

The book review of “Mathematics: A Very Short Introduction” by Timothy Gowers

Mathematics Explained and Clarified

This is a book review of “Mathematics: A Very Short Introduction” by Timothy Gowers [4].

Gowers is a famous [1] British mathematician. He first came to my attention some years ago due to his involvement in the Polymath project [2]. Since then, I occasionally read his blog [3].

The book is written primarily for non-mathematicians. Nevertheless, I was interested to check it out.

At the beginning of the book, in the chapter “Models”, Gowers gives several examples, such as the problem of finding an optimal angle at which to throw an object to maximize how far it will fly before hitting the ground, and the description of properties of gases through moving and colliding particles. These examples belong to physics, not mathematics. While they do involve some mathematical calculations, but so does almost any problem in physics. By itself, it doesn’t make these problems mathematical.

The topic of mathematical models is valid. While probably not deserving an entire chapter, it is worth spending a couple of paragraphs on how mathematical problems arise from the real world. But Gowers fails to show this.

Apart from being contaminated with physics, much worse is that this book is contaminated with philosophy. Effectively admitting this, Gowers even mentions at the end of the book that he was influenced by Wittgenstein:

Anybody who has read this book and the Philosophical Investigations will see how much the later Wittgenstein has influenced my philosophical outlook and in particular my views on the abstract method.

(pp. 139-140)

In my view, an introductory book on mathematics should avoid even philosophy of mathematics, and, most certainly, general philosophy. This shit has no place in mathematics.

Probably, history of mathematics should be avoided as well.

However, I appreciate that Gowers avoided Gödel’s incompleteness theorem. He mentions that he did so on purpose because he views it as an overhyped topic. I completely agree.

The chapter “Numbers and abstraction” is again infested with the philosophical questions of the existence of various types of numbers. Apart from that, the actual introduction of numbers is more or less ok.

The chapter on proofs is good. It gives an example of a proof that can be expanded on each step to give a more detailed argument.

I would expand significantly the chapter “Limits and infinity”. Gowers fails to give the epsilon-delta definition of limit. And much more examples and details should be given. This is one of the most important topics in mathematics and it deserves much more attention in the book.

The next 3 chapters “Dimension”, “Geometry”, “Estimates and approximations” are pretty bad: they are technical and not interesting. I would remove the chapters “Dimension” and “Estimates and approximations” completely.

As for geometry, it certainly deserves a separate chapter in the book, but I would rewrite it completely. Gowers gives too much attention to the 5th postulate about parallel lines, which in my opinion is another overhyped topic, like Gödel’s incompleteness theorem. Instead, I would give some interesting content from plane geometry. And I would even try to avoid the axioms altogether. For example, do you need the axioms to understand the proof (that Gowers does give in this book) of the Pythagorean theorem with 2 ways to count the area? No, you don’t need them.

The section about prime numbers should not be in the chapter “Estimates and approximations”. It makes no sense. Prime numbers certainly deserve a discussion in such a book, but not in a chapter on approximations. It should be in a separate chapter on number theory, and it should be close to the beginning of the book, not at the end of it.

The section on computer science should not be in the chapter “Estimates and approximations”. It makes no sense.

The chapter “Some frequently asked questions” is totally out of place in such a book and I would remove it completely.

Regarding the writing style, I didn’t like that sometimes the book has a condescending tone, which unfortunately is typical for introductory books. An example of this:

Most people think of mathematics as a very clean, exact subject. One learns at school to expect that if a mathematical problem can be stated succinctly, then it will probably have a short answer; often given by a simple formula. Those who continue with mathematics at university level, and particularly those who do research in the subject, soon discover that nothing could be further from the truth.
(p. 112)

On the plus side, the writing is clear and easy to follow, which is actually rare in mathematics. Some chapters are interesting to read even for people with mathematical background, which is not the main target audience. But that should be the case in a good book.

Lastly, a point not about the content of the book, but about the particular edition that I read: the font is too small. The book is pocket sized, but they

tried to put as much on such a small page as in a book of normal size. So, the font is so small that it is really difficult to read. It's really a bad reading experience. I don't know if there are any other editions of this book, but that's the one I got.

Overall, I give this book 3 stars out of 5. After the first half of the book, I was debating with myself between 3 and 4 stars. But the second half of the book was pretty bad, so at the end, I debated with myself between 2 and 3 stars.

References

- [1] https://en.wikipedia.org/wiki/Timothy_Gowers.
- [2] https://en.wikipedia.org/wiki/Polymath_Project.
- [3] <https://gowers.wordpress.com>.
- [4] Timothy Gowers. *Mathematics: A Very Short Introduction*. Oxford University Press, 2002.